Teaching Statement

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In my teaching career, I have found that a math instructor's most important challenge is encouraging students to see challenges as opportunities to learn. For many students, the most important message they need to hear at the beginning of the semester is "math is a skill". On the one hand, this can be liberating: if math is a skill, then like any other skill, anyone can improve their ability through persistent and dedicated practice. On the other hand, this can be frustrating: meaningfully improving a skill requires pushing yourself to do something difficult and uncomfortable. Simply hearing that "math is a skill" is not enough. Students need to be shown that making mistakes in mathematics is a good thing, because mistakes are opportunities to learn. Struggle, and even failure, must be normalized while developing mathematical abilities.

To help my students develop their mathematical skills, I incorporate the following features into my courses:

- A variety of problems that challenge students to find out what they don't yet know;
- Activities that encourage flexible thinking and exploration instead of memorization;
- Credit-based incentives for students to review mistakes on quizzes and assignments;
- Emphasis on the intuition and interpretation behind key results, alongside their implementation.

Using these elements, I treat mathematics as a puzzle to be explored and played with. My hope is to bolster students' confidence and self-efficacy by transforming mathematics into an engrossing exploratory process, and even a playful challenge to solve.

One way an instructor can normalize struggling is by helping students actively identify concepts they may have trouble understanding. The best way for students to identify these concepts is by attempting a variety of problems. I tell all of my students in the beginning of every term, "You don't know what you don't know until you're confronted with the challenge to do it." Some strong students can compute eigenvalues easily, but get stuck when asked to show that a matrix is singular if and only if zero is an eigenvalue. When students get stuck, I tell them, "Excellent! The fact that you're having trouble simply means there's a topic you don't understand yet. Let's find out what that is." Encouraging a growth mindset is important for all levels of learners, from excelling students who want more of a challenge, to students who are still working through previous material. Indeed, when students tell me they feel that they're falling behind, rather than telling them to reread lecture notes or watch recorded lectures, I ask them, "What problems have you attempted?" Occasionally, their answer is "None; I don't know how to do any of them." "That's alright," I say. "Come to my office hours this week, and let's find out what you still have to learn."

Those office hours are vital for students to find where they're having trouble, and it's important to get them in the door. To encourage them to come see me for help, I offer my students an opportunity to correct errors on their quizzes. If they can both demonstrate the correct solutions and explain why their original solutions are incorrect, then I award them partial credit. For example, I had one student try on a quiz to apply Green's Theorem to a non-closed curve in a gradient vector field. When they came to my office to correct their quiz, they had to do more than explain why the fundamental theorem of line integrals is the correct strategy; they had to explain why Green's Theorem is inappropriate. The offer of additional credit helps get them in the door for office hours, but more importantly, examining why a solution is wrong allows students to actively learn from their mistakes and master the interpretation of different mathematical results. In end-of-semester evaluations, students often express appreciation that they are encouraged to learn from their mistakes.

By encouraging exploration instead of memorization, an instructor is more likely to reach students from a wider variety of backgrounds. Studies have shown that active learning practices, such as flipped classrooms and in-class exploration activities, significantly increase retention of knowledge and improve critical thinking skills more generally. I have experimented with a variety of these strategies during my teaching career. When we introduce new results, strategies like these have also helped me focus the discussion on the intuition and geometric and/or physical interpretation of a new result. Centering the discussion on the interpretation of a result, and exploring its implications as a class, promotes conceptual understanding over rote memorization. These best practices in active learning help all students succeed, and as an additional benefit, active learning techniques have been shown to especially promote success among students from underrepresented minorities in STEM. To support success for as many of my students as possible, I aim to provide a safe environment that encourages risk-taking and creative thinking.

When it comes time to solve new problems, creative thinking turns into flexible thinking. I ask students to allow themselves the flexibility to think about solutions analytically, rather than procedurally. Many problems have solutions which may not have a prescribed algorithm (such as solving systems of algebraic equations for Lagrange multipliers), or solutions that come from geometric or physical interpretations (such as integrals in vector calculus). For many introductory math students, such problems require a style of thinking they may not be used to employing in math class. To help them acclimate to this style of thinking, I reassure them, "When you first see a new problem, you will probably not know initially how to solve it. Nor do I expect you to. Allow yourself the flexibility to experiment." It is important for students to hear this message early and often. For example, I recall a conversation with a student who was working on a difficult integration-by-parts problem. As we worked on it, she asked me, "How do I know which component of the integrand to set equal to u and which to set equal to dv?"[1] I told her, "Often you won't know at first. But don't be afraid to try something. If one choice of u and dv doesn't work, you can always try a different choice of u and dv." If students are allowed the space to think flexibly rather than rely on memorization, they often find greater success and enjoyment in their math courses.

In addition to continually improving my teaching strategies for first- and second-year math classes, one of my long-term goals is to lower the barrier of entry to advanced classes like algebraic topology and Lie theory, making them accessible to undergraduate students in math, physics, and data science. It may seem like an ambitious goal, but compare how we teach algebraic topology to how we teach calculus. Calculus is taught at every university in the first year for physics and engineering students as well as math students, typically without introducing the formal definition of limits and continuity. Physicists and engineers don't need to know the formal justification for why derivatives and integrals are well-defined

[1] This question was in reference to the classic integration by parts formula, $\int u\,dv = uv - \int v\,du$.

in order to effectively use the tools of calculus. I am certain that homology and cohomology can be similalry introduced to undergraduates without a formal background in point-set topology. Much of algebraic topology can be discussed by focusing on CW-complexes and simplicial homology and cohomology. This focuses the subject to familiar spaces, avoids a detailed discussion of singular chains, and effectively reduces the prerequisites to abstract algebra. As another example, Lie theory can be made more accessible to an audience without a significant background in differential topology (Brian Hall and John Stillwell each did an excellent job of this in their textbooks on the subject). By focusing on subgroups of matrices, the prerequisites of Lie theory can be reduced to linear and abstract algebra and multivariable calculus. With a brief discussion of manifolds as subsets of $\mathbb{R}^n$, the fundamental aspects of Lie algebras and representation theory can still be explored. In this way, Lie theory and algebraic topology can be made accessible to advanced undergraduates in physics and mathematics, and data science.

By carefully planning discussions that prioritize thinking over knowing, and by giving students the opportunity to self-evaluate and identify where they have difficulty, I have been able to help students overcome the myriad challenges that mathematics presents. When placed in an environment where mistakes are welcomed, students can learn to enjoy mathematics for its spirit of discovery and exploration. In end-of-semester evaluations, one student remarked that they appreciated how I am "always ready to go off on slight tangents if it would help a student's understanding of the material" in responding to students' questions. Beyond effectively teaching the material to my students, fostering an appreciation for the challenge of mathematics is a central goal of mine while teaching. This is a challenge I look forward to at the start of every semester.