Teaching Statement

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1 Introduction: My objectives as an instructor

In my teaching career, I have found that a math instructor's most important challenge is to encourage students to think critically and creatively about solutions to different problems, distinguishing between the *answer* to a problem (i.e., the final "output" of the problem) and the *solution* to a problem (i.e., the process of symbolic and geometric reasoning needed to compute the final answer). Often, creative thinking requires students to begin by acknowledging a simple fact: "I do not yet know how to compute the answer to this problem—and that's okay, because finding how to compute it is part of the solution." My courses are based around encouraging students to actively think about how to arrive at solutions, while I guide them through the process of logical and geometric reasoning. My objective is to demonstrate a central truth about mathematical thinking: Creatively and analytically constructing the solution needed to compute an answer is more important that memorizing specific algorithms and procedures used in computations.

Before students can understand this, however, they must become comfortable with starting from not knowing the solution. Therefore, part of my objective is to encourage students to engage with their uncertainty, and not become frustrated or impatient when they do not immediately see how to solve a problem. I do this by incorporating the following features into my courses:

- A classroom environment where mistakes are welcomed and encouraged;
- Credit-based incentives to review mistakes made on quizzes and assignments;
- Homework that includes "challenge-by-choice" problems to develop broader understanding and identify specific points of confusion;
- Emphasis on the intuition behind key results, in addition to their proper implementation.

Using these elements, I treat mathematics as a puzzle to be explored and played with. From the first day of the semester, I continually communicate to my students that math is a *skill*, not a *trait*, and struggling with a problem simply means there's a concept they don't understand yet, but they will. My hope is to break down the math anxiety some students feel by transforming mathematics into an engrossing exploratory process, and even a playful challenge to solve.

2 My first challenge: normalizing struggle

Students can easily become frustrated when they struggle to understand a particular concept, or have trouble beginning to solve a problem. But starting from a position of conscious uncertainty can be one of the greatest opportunities for learning new mathematics. In her work on mindset theory, psychologist Carol Dweck has argued that making mistakes in mathematics is a good thing, because engaging with mistakes allows students to create new connections and reform their conceptual understanding. For this reason, I challenge my students to *engage with their uncertainty.* This means two things. First, it is an encouragement for students to identify the concepts and types of problems they are struggling with, and to make a plan to improve their understanding. Second, it is an invitation to allow themselves the flexibility to experiment with different solution strategies, as opposed to having specific procedures memorized for different classes of problems.

For many students, the most important message they need to hear at the beginning of the semester is “math is a skill”. On the one hand, this can be liberating: if math is a skill, then like any other skill, anyone can improve their ability by persistent and dedicated practice. On the other hand, this can be frustrating: meaningfully improving a skill requires pushing yourself to do something difficult and uncomfortable. Simply hearing that “math is a skill” is not enough: struggle, and even failure, must be normalized as part of the process of developing mathematical abilities.

One way an instructor can normalize struggling is by helping students actively identify concepts they may have trouble understanding. The most effective way for students to identify these concepts is by attempting a variety of problems. In addition to standard computational exercises, each week I present my students with a variety of challenge problems (such as applications to natural science, or explorations in mathematical curiosities), asking them to choose five from the list to submit (or fewer depending on the number of course credits). In this way, I allow students to select their own challenges and goals week by week. These problems are occasionally more difficult, and some students become discouraged when they can’t immediately solve one of them. When this happens, I remind them, “The fact that this problem is hard for you only means one thing: there is a specific concept that you don’t quite understand yet. But that’s a good thing: there’s a direction for you to grow in, and together, we can find the connections that still need to be made.” After all, students usually don’t know when they don’t understand something until they have to work with it.

I also apply this principle to my assessments. For example, I offer my students an opportunity to correct errors on their quizzes. If they can demonstrate to me that they have learned from their mistakes, and can explain why their original solutions are incorrect, then I award them partial credit. Following my philosophy of framing mistakes as an opportunity to learn, this motivates students to engage with concepts they find particularly difficult. In end-of-semester evaluations, students often express appreciation that they are encouraged to learn from their mistakes.

I have also found that encouraging students to engage with their confusion helps them become more comfortable with taking risks during lecture. In class, I make a habit of thanking students for asking clarifying questions, especially when the question points to a conceptual misunderstanding. Furthermore, when we look at in-class examples, students occasionally suggest an approach that differs from the approach I’ve prepared in my lecture notes. When this happens, I often spend a couple minutes exploring the student’s approach on the blackboard. If we get stuck, we get stuck as a class, and we can unpack the mistake and find where we went wrong.

3 My second challenge: flexible thinking

In addition to identifying and addressing conceptual misunderstandings, the second way I ask students to engage with uncertainty is by allowing themselves the flexibility to think about solutions analytically, rather than algorithmically. Many problems have geometric or physical interpretations that can influence the solution (such as integrals in vector calculus). Others require finding solutions to equations for which there may not be a prescribed algorithm to solve (such as solving systems of algebraic equations for Lagrange multipliers). For skilled mathematicians, the lack of a computational procedure is hardly an obstacle. But for introductory math students, such problems require a style of thinking they may not be used to employing in math class.

One example that occurs early in a student's math education is the difference between derivative and integral calculations. Even for the most convoluted functions, there is a definite procedure for differentiation. Yet there is no universally applicable procedure for computing a closed-form antiderivative (as any probability theorist can confirm). For this reason, computing integrals often requires flexibility: a student must be able to recognize when an integration strategy isn't working, and be willing to try something else.

For students whose mathematical education has been mostly procedural, rather than analytical, this style of thinking may feel unnatural. In one instance, a student was working on a difficult integration-by-parts problem. As we worked on it, she asked me, "How do I know which component [of the integrand] to set equal to u and which to set equal to dv?"[1] I told her, "Often you won't know, at least not immediately. But don't be afraid to experiment. If one choice of u and dv doesn't work, you can always try a different strategy."

Because so much of undergraduate mathematics is based on analytic thinking as much as (if not more than) computational thinking, I make a point in my lectures to prioritize thinking over knowing. If there is a geometric or physical interpretation of a new equation or concept, this interpretation stands at the center of our discussion. I have found that students are much more likely to succeed if they are shown why certain equations *must* be true, and are more than a string of symbols to be memorized. For example, when discussing Green's theorem, Stokes' theorem, and the divergence theorem, I show my students that all vector calculus integration formulas have four common heuristic principles:

(1) The algebraic operations following the $\int$ symbol typically produce a scalar quantity (e.g. dot products can be integrated, but cross products on their own cannot);

(2) The number of integrals, the dimension of the domain of integration, and the number of variables parametrizing the domain (1 for curves, 2 for surfaces, 3 for solids) must all be the same number;

(3) The Jacobian term approximates change in size, and involves derivatives in terms of the same variables that parametrize the domain;

(4) Some calculations can be simplified by either adding or removing an integral, provided they add or remove a "derivative" (i.e., gradient, curl, or divergence) to the integrand.

The first three of these principles are intuitive if students take a moment to understand what integrals represent. The fourth is a summary of Stokes' theorem and the fundamental

[1] This question was in reference to the classic integration by parts formula, $\int u\,dv = uv - \int v\,du$.

theorems of calculus. If students can remember these principles together, and think critically about the domains of integration, they tend to be much more successful with vector field integration problems.

When students have to put analytic thinking into practice on problems, quizzes, or exams, I encourage them to allow themselves the mental space to think about the problem. Occasionally I reassure my students, "When you first see the problems [on the exam or homework], you will probably not know how to solve them. Nor do I expect you to. Allow yourself to analyze the problem before applying theorems from the class." It is important for students to hear this message early and often. There is a myth that to be good at math, you must be able to do calculations and visualize geometric objects purely in your head. Not only is this wrong, but it can lead to students guessing which theorems and procedures to use on a problem before stopping to consider if the procedure applies. If students are shown the absurdity of this myth, they can see the value in analytic thinking, and find greater success and enjoyment in their math courses.

4 Conclusion: What do I want my students to leave with?

By carefully planning discussions that prioritize thinking over knowing; presenting a variety of problems that train both computational skills and abstract concepts; and giving students the opportunity to self-evaluate and identify where they have difficulty, I have been able to help students overcome the myriad challenges that mathematics presents. When placed in an environment where mistakes are welcomed, students can learn to enjoy mathematics for its spirit of discovery and exploration. In end-of-semester evaluations, one student remarked that they appreciated how I am "always ready to go off on slight tangents if it would help a student's understanding of the material" in responding to students' questions. Beyond effectively teaching the raw material to my students, fostering an appreciation for the challenge of mathematics is a central goal of mine while teaching. This is a challenge I look forward to at the start of every semester.