Teaching Statement

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Mathematical literacy is a skill that lies at the core of many major careers in both the public and private sector. In spite of this, a disturbing number of individuals suffer from "math anxiety." Over the course of my teaching at Penn State, I have found that one source of students' frustration with mathematics comes from the tendency that math exercises are based on finding the one right answer. By contrast, students are seldom shown the value of the thinking process. Unfortunately, overemphasizing correctness of the final result has led many students to attempt to memorize different kinds of problems, rather than deeply understand why certain techniques are used. Additionally, it widens the disconnect between algebraic manipulations and computations, and the tangible numerical or geometric significance these manipulations represent.

Accordingly, my courses are based around encouraging students to actively think about how to arrive at a solution, while I guide them through the process of mathematical and logical reasoning. My objective is to demonstrate a central truth about mathematical thinking: the most productive way to approach a challenge is by thinking about what procedures are appropriate for a particular problem and in a particular context, not by simply knowing how to perform different procedures. I pursue this objective by incorporating the following features into my courses:

- An environment where mistakes in class and in office hours are welcomed and encouraged to facilitate active group learning;
- Credit-based incentives to address and correct mistakes made on past quizzes and assignments;
- Emphasis on the intuition and larger connections of key results in addition to their proper use, often through the use of graphical and scientific software;
- Homework consisting of simple nightly exercises to train basic concepts, and more challenging weekly problems to develop broader skill and understanding.

Using these elements, I treat mathematics as a puzzle to be explored and played with. From the first day of the semester, I continually communicate to my students that there is no such thing as a "math person;" instead, mathematics is based around logic and reason, and logic and reason are central to what make us human. My hope is that in this way I can break down the math anxiety some students may have by transforming mathematics into an engrossing exploratory process.

When I prepare for class, I write my lectures in a way that prioritizes thinking over knowing, such as by emphasizing the geometric or physical interpretation of a new equation or concept. I have found that students are much more likely to succeed if they are shown why certain equations *must* be true, and are more than a string of symbols to be memorized. For example, when students learn about Green's theorem, Stokes' theorem, and the divergence theorem, I show my students that all of the integration formulas for vector fields and scalar functions have four common heuristic principles:

(1) The algebraic operations following the $\int$ symbol typically produce a scalar quantity (e.g. dot products can be integrated, but cross products on their own cannot);

(2) The number of integrals, the number of variables in the surface or curve parametrization, and the dimension of the domain of integration (1 for curves, 2 for surfaces, 3 for solids) must all be the same number;

(3) The Jacobian term approximates change in size, and involves derivatives in terms of the same variables that parametrize the domain (such as a cross product of two different derivatives with respect to the variables parametrizing a surface);

(4) One can sometimes simplify calculations by either adding or removing an integral, provided they add or remove a "derivative" (such as gradient, curl, or divergence) to the integrand.

The first three of these principles are intuitive if students take a moment to understand what integrals represent. The fourth is a summary of Stokes' theorem and the fundamental theorem of calculus. If students can remember these principles together, and think critically about the domains of integration, they tend to be much more successful with vector field integration problems.

In-class participation is a critical component of the classes I teach. Whenever possible, I use active learning techniques to teach new concepts, such as worksheets and group activities. For example, in Calculus 1, the concept of derivatives can be introduced by having students explore how to approximate a rate of change with increasing precision. Guided with in-class activities, students can independently construct secant lines. As a class, students can then arrive at the limit definition of a derivative. Active learning strategies such as this reinforce the intuition behind a new concept, rather than encourage memorizing an algebraic statement.

In her work on mindset theory, psychologist Carol Dweck has argued that making mistakes in mathematics is a good thing, because engaging with mistakes allows students to create new connections and reform their conceptual understanding. One benefit of having students independently work out parts of an argument in class is it encourages comfort with making mistakes. I have had students suggest an approach to solving an example problem in class that differed from the approach I'd prepared in my lecture notes. When this happens, I spend a couple minutes exploring the student's approach on the blackboard. If we get stuck, we get stuck as a class, and are able to unpack the problem-solving process together. By encouraging mistakes in class, an instructor can break down the sentiment that math is purely about problem-solving and finding the one right answer, and demonstrate that instead math is about curiosity and logical exploration.

An equally important aspect of mathematics education is putting what one learns in lecture or from the textbook into practice. In intermediate math courses, I assign a variety of problems that fall into three categories: Exercises, Problem Sets, and Quizzes. Each type of assignment is structured differently, and trains students in different ways with specific goals based on Bloom's taxonomy of educational objectives.

Exercises are assigned in sets of three to five after each class, which are simple or routine calculations that reflect what we did in that specific class day (such as using row reduction to find the rank of a matrix after defining rank in class). My objective here is to encourage students to put the basics of what they have learned in class into immediate practice, helping them to remember and understand the concepts covered on that day (covering the first two levels of Bloom's taxonomy).

Problem Sets, on the other hand, are assigned weekly, and are significantly more challenging than routine calculations. These more sophisticated assignments have students work through proving a result from class independently (such as the integrating factor formula for first-order linear ordinary differential equations), exploring concrete applications of principles from class (such as using mathematical models to unpack the significance of the multivariable chain rule), or solving purely mathematical problems whose solutions are synthesized from several concepts (such as proving $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$ using polar coordinates). Unlike Exercises, these are not intended to be used to practice what they have learned, and cannot be solved simply by remembering results. Rather, the Problem Sets allow students to identify concepts they may not yet understand and learn new math in the context of actively solving a problem. With these assignments, I challenge students to apply and analyze concepts in different scenarios, and even evaluate proposed solutions and create new ones (covering the third through sixth levels of Bloom's taxonomy). In end-of-semester teaching evaluations and in personal correspondences, my former students have written that they "appreciated the problem sets, because they helped me understand the concepts at a deeper level than the questions on the exam", and that they "always left office hours feeling a sense of hopeful progress and the joy of having tackled difficult math problems." Although students find these problems more difficult, several have told me they understand the material much more thoroughly than they'd expected they would, and in some cases have expressed appreciation for the excitement of the added challenges.

Quizzes are given out in class at the end of every week. The quiz problems are more sophisticated than routine exercises, but less demanding than the challenging problem sets, and cover the material from the previous week. Rather than solely using these quizzes for myself to evaluate their understanding, I want students to use these quizzes to evaluate themselves and pinpoint where they get confused. For this reason, I incentivize my students to address these areas of confusion. After each quiz, they have the opportunity to bring the quiz to me in office hours and explain both why their answers to certain questions are incorrect, and how to arrive at the correct answer. If they can do this, then I give them half of the points back that were taken off. Students have expressed appreciation for this practice; one reflected in an end-of-semester evaluation that "going over our quizzes really showed what went wrong and exactly why it did." Following my philosophy of framing mistakes as an opportunity to learn, this motivates students to see where they went wrong and how to think critically about different problems.

In addition to going over quizzes, office hours are critical for students to discuss the challenging problems I assign. I make an effort to be as available to my students as possible, whether this is by meeting students in-person during office hours or outside appointments, or by answering questions via email or online class forums. When working with students one-on-one, I maintain the same philosophy that I do in lecture of leading students through the problem-solving process with guiding questions. Often their responses provide insight into where their difficulty in understanding originates. In these circumstances, I frequently help them learn how to look up important theorems and examples in the textbook on their own, and break down how a theorem or concept may apply to a problem on their homework. One student remarked in their end-of-semester evaluation that they found office hours to be "incredibly useful ... [because the instructor] worked through problems with me instead of just telling me how to do them." In the more personal setting of office hours, students take a more active role in thinking through their difficulties. For example, while talking with a calculus student having trouble with analytic geometry, I discovered this student did not understand that an algebraic equation such as $x^2 + y^2 + z^2 = 1$ corresponds to a surface in 3-dimensional space by describing the coordinates of points on that surface. Once I found the source of his trouble, he was able to address this confusion and work out the solutions to several other problems on his own.

I am also a strong believer in using technology to assist in the learning process both in and outside of class. For many lectures, I prepare a collection of digital images and animations of geometric concepts using MATLAB and Geogebra. These dynamical illustrations are especially useful when used to supplement examples in differential equations and vector calculus. For example, they make it easier to demonstrate level curves of multivariable functions by comparing level curves with the intersection of a surface graph with a horizontal plane in real time. I have also found graphical software to be a powerful and engaging exploratory tool. Several of the Problems I assign to my students involve using graphical software to explore properties of strange multivariable functions or stability of solutions to systems of differential equations. Additionally, I keep an up-to-date course website with the syllabus, assignments, quizzes and quiz solutions, and animations of some geometric concepts when applicable. I also regularly upload my own lecture notes to the course website for students to study from. My course website also features a discussion forum, where students can post questions on the homework and I can answer them online. I have found that this discussion forum is very helpful to prepare me for different kinds of questions and misconceptions during office hours.

Through carefully planning lectures and guided class discussions centered on thinking through new ideas, presenting a variety of homework problems exercising both computational skills and abstract concepts, and providing students with regular opportunities to self-evaluate and identify where they have difficulty, I have been able to help students overcome the myriad challenges that mathematics presents. By creating a classroom culture where mistakes are welcomed, I have found that students can learn to enjoy mathematics for its spirit of discovery and exploration. In end-of-semester evaluations, one student remarked that they appreciated how I am "always ready to go off on slight tangents if it would help a student's understanding of the material" in responding to students' questions. Beyond effectively teaching the raw material to my students, fostering an appreciation for the challenge of mathematics is a central goal of mine while teaching. This is a challenge I look forward to at the start of every semester.